GAS VELOCITY PROFILE IN THE TURBULENT FLOW OF A GAS SUSPENSION IN A CHANNEL OF ANNULAR CROSS SECTION

F. E. Spokoinyi

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On the basis of concepts concerning two dynamically independently acting zones of a stream in an annular channel and the existence of a universal velocity profile in each zone, the parameters of this profile are calculated for the carrying medium of the gas-suspension stream.

Channels of annular cross section find the most extensive distribution in different types of heat exchangers, including those using a gas-suspension stream as the heat-transfer agent. Such channels are of interest for the heat-removal systems of various reactors, pneumatic transport systems, and other technological devices. The study of the heat exchange of gas-disperse systems having such surfaces is paid undeservedly little attention in comparison with gas-suspension streams in pipes [1]. At the same time, the available reports [2-4] have a purely empirical nature. The reason for the lagging of theoretical studies is the total absence of data on the characteristics of the average velocity profile of the carrying medium in the flow of a gas suspension in an annular channel. In the calculation of the velocity profiles of homogeneous streams in such channels [5-8] the stream is divided into two coaxial zones (from r_1 to r_* and from r_* to r_2) regardless of the type of dependence used for the turbulent viscosity (Deissler's or van Driest's equations or a "central law"). The velocity profile in each zone is determined by the proximity of only one of the channel walls. The conditions $\tau = 0$ and $dv/dr = 0$ are usually used to separate the zones and determine the value of r_*. An analysis of the known experimental data indicates the noncoincidence of these boundaries and leads the authors of [8], for example, to the conclusion that there is negative turbulent viscosity. The boundary corresponding to the condition $\tau = 0$ lies at the intersection of the extensions of the velocity profiles calculated for each zone [8]. The method proposed by Maubach [9, 10] for homogeneous streams seems the most convenient for analysis and at the same time is accurate enough. The essence of the method consists in the use of balance equations based on the assumption that a universal logarithmic velocity distribution exists in both zones of an annular channel. The results of calculations of homogeneous streams by this method [9, 10] agree well with the experimental data. The introduction of particles into the stream leads to a change in the profile of the carrying medium. In this case the turbulent mechanism of momentum transfer is retained in the core of the stream while in the boundary zones the momentum transfer in the carrying medium takes place primarily through molecular viscosity. Such considerations allow one to assume that in the movement of a gas suspension, just as in the flow of a homogeneous medium, the gas velocity profile in the zones of the channel has a logarithmic nature in the core of the stream and a linear nature near the wall. In this case the parameters of these functions vary considerably in the presence of particles in the stream. A hydrodynamic theory of the heat exchange of gas-suspension streams in tubes developed in [11] on the basis of a similar assumption was confirmed by experimental data. Thus, the stabilized stream of a gas suspension in an annular channel can be conditionally separated into an inner zone (index 1) and an outer zone (index 2). The velocity profile in each of the zones can be represented on the basis of the two-layer model in the following form:

viscous layer

$$v_i^+ = y_i^+; \quad 0 \leqslant y_i^+ \leqslant \delta^+, \tag{1}$$

turbulent core

$$v_i^+ = \frac{1}{\chi} \ln y_i^+ + a; \quad \delta^+ \leqslant y_i^+ \leqslant y_i^+ (r = r_*). \tag{2}$$

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In these expressions $v_i^+ \equiv v_i/v_i^*$; $y_i^+ = v_i^* |r - r_i|/\nu$; $v_i^* = \sqrt{\tau_{wi}/\rho}$; χ and a are constants of the logarithmic profile. For a continuous stream the Karman constant is $\chi_0 = 0.4$ and the value of $a \equiv (1/\chi)\ln\delta^+ + \delta^+$, determined from the condition of continuity of the velocity profile at $y_i^+ = \delta^+$, is 5.5. In the determination of the parameters of profiles (1) and (2) for a gas-suspension stream, it is necessary to consider that this profile corresponds to the shear stresses produced only by the viscous and turbulent friction of the carrying medium. To find the values of these stresses at the channel walls (τ_{wi}) we use the method proposed in [12] for gas-suspension streams in round pipes. This method is more reliable than that developed in [13], since the series of insufficiently grounded assumptions adopted in [13] often leads to physically unjustified results (for example, with $\mu > 3$ and $\rho_s/\rho \simeq 10^3$, the thickness of the boundary layer turns out to have an imaginary value according to [13]). According to [12] the pressure losses to gas friction are determined from the total losses after subtraction of the contribution of the impact interaction of the particles with the wall (the expenditures in raising the particles are negligibly small under these conditions). For the impact frequency, the pressure losses, and the coefficient of resistance due to the shock interaction one can obtain

$$n_i = \frac{(r_2^2 - r_1^2)N}{2r_i} \quad \frac{v_s'}{2(r_2 - r_1)}, \tag{3}$$

$$\frac{dP_{im}}{dx} = -\frac{(2\pi r_1 n_1 + 2\pi r_2 n_2)\, m_s v_{sw} \zeta}{\pi (r_2^2 - r_1^2)} = -\frac{m_s N v_s' v_{sw} \zeta}{r_2 - r_1}, \tag{4}$$

$$\xi_{im} = -\frac{2D}{\rho \bar{v}^2}\left(\frac{dP_{im}}{dx}\right) = 4\zeta\mu\varphi' \frac{v'}{v_0'} \sqrt{\frac{\xi_0}{8}}\,\frac{v_{sw}}{v_s}, \tag{5}$$

where ζ is the coefficient of velocity loss upon impact, $\varphi' = v_s'/v'$ characterizes the degree of entrainment of the particles in the pulsation motion of the gas and can be estimated from [14]; v'/v_0' reflects the countereffect of the particles on the intensity of the pulsations of the carrying medium [15, 16]. For particles with $r_s \gg \delta$ and $r_s \ll \delta$ the following respective equations can be obtained ($\xi_f = 2\Delta P_f D/L\rho\bar{v}^2$):

$$\xi = \xi_f - \sqrt{2\xi_0}\,\zeta\varphi' \frac{v'}{v_0'}\mu, \tag{6}$$

$$\xi = \xi_f\left[1 + \frac{\zeta\varphi' \mathrm{Re}}{8}\,\frac{v'}{v_0'}\,\frac{d_s}{D}\sqrt{2\xi_0}\,\mu\right]^{-1}. \tag{7}$$

These functions allow one to calculate the characteristic frictional stresses for the entire stream in an annular channel. To determine the corresponding values in the zones one can, as in [10], use the equality of the pressure losses to friction in the zones and in the entire stream. In this case $\tau_w/D = \tau_{wi}/D_i$, where D_i are the hydraulic diameters of the zones. With allowance for the definitions $\alpha \equiv r_1/r_2$ and $\beta \equiv r_*/r_2$ one can obtain by analogy with [9]

$$\frac{\tau_{w1}}{\tau_w} = \frac{\beta^2 - \alpha^2}{\alpha(1-\alpha)}; \quad \frac{\tau_{w2}}{\tau_w} = \frac{1-\beta^2}{1-\alpha}; \quad \frac{\tau_{w1}}{\tau_{w2}} = \frac{\beta^2 - \alpha^2}{\alpha(1-\beta^2)}. \tag{8}$$

Thus, the following values in (1)-(2) are unknown: a (or δ^+), χ, and r_* (or β). One of the conditions relating these parameters is the intersection of the velocity profiles at the boundary of separation of the zones, i.e., at the point $r = r_*$: $v_{max} = v_1^* v_{1max}^+ = v_2^* v_{2max}^+$. By determining the v_i^* for the gas-suspension stream from the values of τ_{wi} and ξ from (6)-(8), we obtain

$$\sqrt{\frac{\tau_{w1}}{\rho}}\left[\frac{1}{\chi}\ln\left(\frac{r_* - r_1}{\nu}\sqrt{\frac{\tau_{w1}}{\rho}}\right) + a\right] = \sqrt{\frac{\tau_{w2}}{\rho}}\left[\frac{1}{\chi}\ln\left(\frac{r_2 - r_*}{\nu}\sqrt{\frac{\tau_{w2}}{\rho}}\right) + a\right]. \tag{9}$$

It follows from Eqs. (8) and the equality $r_2/\nu\sqrt{\tau_w/\rho} = \mathrm{Re}/2(1-\alpha)\sqrt{\xi/8}$ that

$$\begin{aligned} v_{1max}^+ &= \frac{1}{\chi}\ln\left[\frac{\beta-\alpha}{2(1-\alpha)}\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\alpha)}}\,\mathrm{Re}\sqrt{\frac{\xi}{8}}\right] + a; \\ v_{2max}^+ &= \frac{1}{\chi}\ln\left[\frac{1-\beta}{2(1-\alpha)}\sqrt{\frac{1-\beta^2}{1-\alpha}}\,\mathrm{Re}\sqrt{\frac{\xi}{8}}\right] + a. \end{aligned} \tag{10}$$

Then Eq. (9) can be written in the form

$$\sqrt{\frac{\alpha(1-\beta^2)}{\beta^2-\alpha^2}} = \frac{\frac{1}{\chi}\ln\left[\frac{\beta-\alpha}{2(1-\alpha)}\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\alpha)}}\,\mathrm{Re}\sqrt{\frac{\xi}{8}}\right] + a}{\frac{1}{\chi}\ln\left[\frac{1-\beta}{2(1-\alpha)}\sqrt{\frac{1-\beta^2}{1-\alpha}}\,\mathrm{Re}\sqrt{\frac{\xi}{8}}\right] + a}. \tag{11}$$

The second condition is the equation of mass balance of the gas stream with allowance for its division into zones:

$$\bar{v}_1 F_1 \varepsilon_1 + \bar{v}_2 F_2 \varepsilon_2 = \bar{v}\,(F_1 \varepsilon_1 + F_2 \varepsilon_2),$$

where F_i is the cross-sectional area of the i-th zone; ε is the average porosity in the zone; $\bar{v}_i$ is the velocity of the continuous component of the gas-suspension stream averaged over the cross section of the zone. When $\varepsilon_1 = \varepsilon_2$ this equation can be represented in the following dimensionless form with allowance for Eq. (8):

$$\frac{\bar{v}_1}{v_1^*}\,\frac{\beta^2-\alpha^2}{1-\alpha^2}\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\alpha)}} + \frac{\bar{v}_2}{v_2^*}\,\frac{1-\beta^2}{1-\alpha^2}\sqrt{\frac{1-\beta^2}{1-\alpha}} = \sqrt{\frac{8}{\xi}}. \tag{12}$$

In order to use this condition we determine the values of the average dimensionless velocities in each zone ($R \equiv r/r_2$; $\Delta_i \equiv \delta_i/r_2$):

$$\frac{\bar{v}_1}{v_1^*} = \frac{2}{\beta^2-\alpha^2}\int_\alpha^\beta v_1^+ R\,dR = \frac{1}{\chi}\ln\frac{\beta-\alpha}{\Delta_1} + \left[\frac{v_1^* r_2}{\nu} + \frac{2\alpha}{\chi(\beta^2-\alpha^2)}\right]\Delta_1 - \frac{3\alpha+\beta}{2\chi(\alpha+\beta)} + O(\Delta_1^2), \tag{13}$$

$$\frac{\bar{v}_2}{v_2^*} = \frac{2}{1-\beta^2}\int_\beta^1 v_2^+ R\,dR = \frac{1}{\chi}\ln\frac{1-\beta}{\Delta_2} + \left[\frac{v_2^* r_2}{\nu} + \frac{2}{\chi(1-\beta^2)}\right]\Delta_2 - \frac{3+\beta}{2\chi(1+\beta)} + O(\Delta_2^2). \tag{14}$$

If one neglects terms of order Δ^2 and the second terms in the brackets of Eqs. (13) and (14) because of their smallness in comparison with the first terms and if one uses the expression for a through δ^+, then (13) and (14) can be reduced to the following form:

$$\frac{\bar{v}_1}{v_1^*} \simeq \frac{1}{\chi}\ln\frac{v_1^* r_2(\beta-\alpha)}{\nu} + a - \frac{3\alpha+\beta}{2\chi(\alpha+\beta)} = v_{1\max}^+ - \frac{3\alpha+\beta}{2\chi(\alpha+\beta)}, \tag{15}$$

$$\frac{\bar{v}_2}{v_2^*} \simeq \frac{1}{\chi}\ln\frac{v_2^* r_2(1-\beta)}{\nu} + a - \frac{3+\beta}{2\chi(1+\beta)} = v_{2\max}^+ - \frac{3+\beta}{2\chi(1+\beta)}. \tag{16}$$

When $\chi = \chi_0 = 0.4$ these expressions coincide with those obtained in [9] for a continuous stream in an annular channel. The function obtained in [12] for a gas-suspension stream in a round tube follows from (16) as $\beta \to 0$. If one substitutes the functions (15) and (16) into (12) with allowance for Eqs. (10), then the equation obtained can be solved jointly with Eq. (11) to determine the parameters of the velocity profile. The dependence of the Karman constant on the particle concentration, obtained in [17] for gas-suspension streams in round pipes, can be taken as the third equation in this system. This empirical dependence is well approximated by the equation [12]

$$\chi/\chi_0 = 1 + 0.16\,\mu^{0.9}. \tag{17}$$

Equation (17) can be used only by way of a first approximation, since it is obtained for a velocity profile which differs from (1)-(2) and does not take into account the properties of the dispersing agent. By transforming Eq. (11) with allowance for (10) one can express the value $v_{1\max}^+$ through β:

$$v_{1\max}^+ = \left\{\frac{1}{\chi}\ln\left[\frac{1-\beta}{\beta-\alpha}\sqrt{\frac{\alpha(1-\beta^2)}{\beta^2-\alpha^2}}\right]\right\}\Big/\left[\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\beta^2)}} - 1\right]. \tag{18}$$

Substituting this expression into (15) and (16) and then into (12) we obtain the following equation:

$$\sqrt{\frac{1-\beta^2}{1-\alpha}}\left\{\frac{\ln\left[\frac{1-\beta}{\beta-\alpha}\sqrt{\frac{\alpha(1-\beta^2)}{\beta^2-\alpha^2}}\right]}{1-\sqrt{\frac{\alpha(1-\beta^2)}{\beta^2-\alpha^2}}} - \frac{(3+\beta)(1-\beta)-(3\alpha+\beta)(\beta-\alpha)\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\beta^2)}}}{2(1-\alpha^2)}\right\} = \chi\sqrt{\frac{8}{\xi}}. \tag{19}$$

The solution of (19) allows one to establish the dependence of the position of the boundary β between the zones on the ratio α of the radii of the walls of the annular channel and on the value of the complex $\chi\sqrt{8/\xi}$, which takes into account the effect of the concentration μ. The results of calculations by (19) performed by G. V. Derevyanko are presented in Table 1. As follows from Table 1, the values of it essentially depend on α, decreasing slightly with an increase in $\chi\sqrt{8/\xi}$ and being almost independent of it at large α. The results of the calculations of [9] for a homogeneous stream are in complete agreement with the data of Table 1 on the part of the dependence of β on α and the Reynolds number. In this connection, the

TABLE 1. Dependence of the Values of β and $a\chi + \ln(\mathrm{Re}\sqrt{\xi/8})$ (the latter are given in parentheses) on the Geometry of the Channel and the Complex $\chi\sqrt{8/\xi}$

$\chi\sqrt{8/\xi}$	α 0,1	0,2	0,4	0,6	0,8
6	0,3494 (8,30)	0,4727 (8,33)	0,6452 (8,35)	0,7796 (8,39)	0,8955 (8,43)
10	0,3392 (12,30)	0,4651 (12,35)	0,6414 (12,40)	0,7782 (12,38)	0,8952 (12,47)
14	0,3338 (16,30)	0,4610 (16,33)	0,6394 (16,36)	0,7773 (16,39)	0,8950 (16,42)
18	0,3305 (20,32)	0,4584 (20,34)	0,6381 (20,40)	0,7768 (20,31)	0,8949 (20,30)
22	0,3282 (24,34)	0,4567 (24,34)	0,6373 (24,31)	0,7765 (24,63)	0,8948 (25,62)
26	0,3265 (28,33)	0,4554 (28,33)	0,6366 (28,42)	0,7762 (28,38)	0,8948 (27,78)
30	0,3253 (32,33)	0,4544 (32,40)	0,6361 (32,46)	0,7760 (32,83)	0,8947 (36,76)

coefficients of resistance were determined in [9, 10] for a homogeneous stream since in that case, in contrast to the problem under consideration, the values $\chi = \chi_0$ and $a = a_0$ are known. Having available the values of β, one can on the basis of Eqs. (10) and (18) obtain an expression for calculating a:

$$a\chi + \ln\left(\mathrm{Re}\sqrt{\frac{\xi}{8}}\right) = \left[\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\beta^2)}} - 1\right]^{-1} \ln\left[\frac{1-\beta}{\beta-\alpha}\sqrt{\frac{\alpha(1-\beta^2)}{\beta^2-\alpha^2}}\right] - \ln\left[\frac{\beta-\alpha}{2(1-\alpha)}\sqrt{\frac{\beta^2-\alpha^2}{\alpha(1-\alpha)}}\right]. \tag{20}$$

The results of calculations of $a\chi + \ln(\mathrm{Re}\sqrt{\xi/8})$, which take into account the effect of the particles on the velocity profile, are also presented in Table 1. It is almost independent of the value of α. This is evidently connected with the fact that a and χ are universal parameters of the velocity profile and depend only on the properties of the particles and their concentration in the stream. The data of the calculations by (20) are approximately described by a linear function of $\chi\sqrt{8/\xi}$:

$$a\chi + \ln(\mathrm{Re}\sqrt{\xi/8}) \simeq 2.3 + \chi\sqrt{\frac{8}{\xi}}. \tag{21}$$

This function, obtained for channels of annular cross section, corresponds to a relation brought out earlier for gas-suspension streams in pipes [12]. With $\chi = \chi_0 = 0.4$ and $a = a_0 = 5.5$ the expression (21) satisfies the condition of a limiting process, since it agrees with high accuracy with the well-known data of [18] on the resistance of homogeneous streams ($\mu = 0$). The function (21) makes it possible, by estimating beforehand the values of ξ and χ from Eqs. (6), (7), and (17) for the given Re, α, and μ and with the properties of the particles and the flow resistance ξ_f being known, to determine the value of a. The dependence of the Karman constant in a gas-suspension stream on the concentration and the characteristics of the particles can be refined on the basis of the expression (21). For this one must have available data on the pressure losses in a gas-suspension stream for the same particles at different values of μ and the Reynolds number. Then, by solving a system of equations of the type of (21) for a fixed value of μ and different Re, one can establish the desired functions for the parameters χ and a of the profile.

If, as in the calculation of the heat-exchange intensity [1], the determination of the thickness of the viscous layer in the gas-suspension stream is of the greatest interest, the value of δ^+ can be obtained through a numerical or even a graphic solution of the equation $\ln\delta^+ = \chi(\delta^+ - a)$. The absolute values of δ at both walls of the channel are calculated using (8) on the basis of the equation

$$\delta_i = \frac{2r_2(1-\alpha)\delta^+}{\mathrm{Re}\sqrt{\xi/8}}\sqrt{\frac{\tau_w}{\tau_{wi}}}. \tag{22}$$

Reliable experimental data on the velocity profile of the carrying medium of a gas-suspension stream in an annular channel are absent at present, which can be explained by certain experimental difficulties which arise in this case. At the same time, the obtaining of this kind of data is very urgent, just for the testing of the proposed method of calculation through the comparison of experimental and calculated data on heat exchange with such streams.

NOTATION

D, Equivalent diameter, m; m_s, mass of one particle, kg; N, calculating concentration of particles, $1/m^3$; v, v*, absolute and dynamic velocities, m/sec; r_1, r_2, r_*, radii of walls of annular channel and of boundary separating the zones, m; y, distance from wall, m; δ, thickness of viscous layer, m; ξ, coefficient of resistance; μ, flow-rate mass concentration; ρ, density, kg/m^3; τ, shear stresses, N/m^2. Indices: s, solid particles; O, stream without particles; ', pulsation value; w, value at wall; 1, 2, inner and outer walls and zones adjacent to them.

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